

20.8: The Nature of the Roots

of a quadratic

★ Recall: the solutions (roots) of an equation are the zeros (x-intercepts) of its graph.

- a Parabola is the graph of a quadratic

The Discriminant: $B^2 - 4AC$
(part of the quadratic formula
under the radical)

* by computing the discriminant
($b^2 - 4ac$) you can describe the
roots of any quadratic.

* the following table must be
memorized!!

$b^2 - 4ac$	Roots	Graph
1) < 0 (negative)	1) Imaginary	1) Does not intersect x-axis

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2) $= 0$	2) real, rational, equal	2) tangent to x-axis
3)a) > 0 , perfect sq.	a) real, rational, unequal	a) 2 x-intercepts
3)b) > 0 , not perfect	b) real, irrational, unequal	b) 2 x-intercepts

Basically:

1) if discriminant is (+) - roots are \mathbb{R}
(-) - roots are imaginary

2) if discriminant is Perfect - rational
not Perfect - irrational

3) if discriminant = 0, equal roots

Ex: Compute the discriminant and describe the roots of the following quadratics:

(a) $x^2 - 2x = -5$

(b) $x^2 - 2x - 1 = 0$

Σx :

The graph of $y = -x^2 - 6$

(1) is tangent to the x -axis
(3) lies entirely above the x -axis

(2) intersects the x -axis at two points
(4) lies entirely below the x -axis

EXERCISES WITH OPEN-RESPONSE PROBLEMS

In 1–12, for each equation: a. Evaluate the discriminant $b^2 - 4ac$. b. Using the value of $b^2 - 4ac$, select the choice below that describes the nature of the roots of the equation:

- (1) real, rational, and unequal
- (2) real, rational, and equal
- (3) real, irrational, and unequal
- (4) imaginary

1. $x^2 + 3x - 5 = 0$

4. $x^2 - 10x + 25 = 0$

7. $x^2 = 7x - 6$

10. $x^2 + 9 = 2x^2 + x$

2. $x^2 + 3x - 10 = 0$

5. $x^2 + 8x + 17 = 0$

8. $2x^2 + 3x = 4$

11. $x - 8 = x^2 - 2x$

3. $x^2 + 3x + 4 = 0$

6. $x^2 + 7x - 30 = 0$

9. $4x^2 + 9 = 12x$

12. $x^2 + 6 = 3x^2 + x$

In 13–21, in each case, select the choice below that describes the graph of the parabola:

- (1) It is tangent to the x -axis.
- (2) It intersects the x -axis at 2 points.
- (3) It lies entirely above the x -axis.
- (4) It lies entirely below the x -axis.

13. $y = x^2 - 2x - 8$

16. $y = x^2 - 6$

19. $y = 10x^2$

14. $y = x^2 - 2x + 1$

17. $y = 2x^2 - 5x + 4$

20. $y = -x^2 + 3x - 7$

15. $y = x^2 + 3$

18. $y = -x^2 - 10$

21. $y = 12 - x^2$

In 22–33, in each case, solve the quadratic equation, and check both roots.

22. $x^2 - 6x + 5 = 0$

25. $x^2 = 2x - 10$

28. $9x^2 + 4 = 0$

31. $x - x^2 = \frac{1}{4}$

23. $x^2 - 6x + 13 = 0$

26. $x^2 + 20x = 6x - 49$

29. $3x^2 + 11x = 4$

32. $\frac{x^2}{2} = x - 2$

24. $x^2 = 2x + 6$

27. $x^2 + 4x = x$

30. $x^2 + 10x + 26 = 0$

33. $\frac{x-2}{2} = \frac{x-1}{x}$

Pg 953: 3-33 (:8)

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18. $y = -x^2 - 10$

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33. $\frac{x-2}{2} = \frac{x-1}{x}$